Lesson Research Proposal for 6th Year Higher Level Financial Maths

Date of lesson: 07/02/2019
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1. Title of the Lesson: A Journey in Finance

2. Brief description of the lesson

Students will investigate the cost involved in purchasing a car in three years through the creation of a series of payments. From this it is hoped that students will identify correctly the number of payments involved in a sum, hence, identifying the creation of a geometric series. This lesson will equip students in creating a timeline for any given payment over a period of time, eliminating rote learning off of 'savings=multiply, loans=division' whilst removing the fear and anxiety around financial maths as a topic.

3. Research Theme

In St Mary's College the Teachers in the Maths department are committed to:

- Engaging in professional development and professional collaboration
- Working together to devise learning opportunities for students across and beyond the curriculum
- Collectively developing and implementing consistent and dependable formative and summative assessment practices
- Ensuring that our students experience opportunities to develop the skills and attitudes necessary for lifelong learning

4. Background & Rationale

We chose this topic as we have found that students often do not associate the area of geometric sequences covered in patterns and sequences when they are studying financial mathematics. Students experience lots difficulty creating the series and this often boils down to trying to learn just the procedures, such as: savings = multiply and loans = divide.

Teachers have noticed another area that causes difficulty for students is creating the timeline for the series of payments dependent on payments at the beginning or end of a month, year etc. In Section B of the exam students fear and show misconceptions when dealing with a change throughout the period of time or a gap in payments.

The maths team consider that if we created a series of lessons which incorporate the financial applications with patterns and sequences, then we would increase students' understanding and their ability to tackle problems without resorting to the procedures alone and in addition, we think it would reduce the overall time required to teach the topic.

This lesson is aimed at Leaving Certificate Higher Level students.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	***Learning outcomes for this unit***	***Related later learning outcomes***
Students will have used the formulae for geometric sequences and series to solve problems "recognise whether a sequence is arithmetic, geometric or neither" "investigate geometric sequences and series'" Students will be familiar with compound interest and its uses in savings and loans. "*compound interest, depreciation (reducing balance method), income tax and net pay (including other deductions)"*	Students will expand their learning of loans including the use of present value. *"use present value when solving problems involving loan repayments and investments*" Students will apply their knowledge of Geometric series *"solve problems involving finite and infinite geometric series including applications such as recurring decimals and financial applications, e.g. deriving the formula for a mortgage repayment"*	The formulisation of the Amortisation formula *"Ways to express a general relationship arising from a pattern or context."* University for those students who go on to take business courses. Roughly 30% of our students go on to study Business.

6. Goals of the Unit

- Students will work easily between APR/AER and the monthly equivalent rate
- Students will distinguish between present value and future value of a number of payments
- Students will use their knowledge of geometric series to identify an efficient way to calculate monthly loan and/or savings repayments
- Students can explain the difference in a geometric series for the value of future payments and the present value of a series of future payments
- Students will use their knowledge of geometric series to help them make financially beneficial decisions

7. Unit Plan

How the research lesson fits into the larger unit plan, helping to show the bigger picture of the whole unit and the progression of learning. Clarify where the research lesson will be taught.

Lesson	Brief overview of lessons in unit
1	Converting APR to Monthly Equivalent Rate
2	Review/Introduction to Compound Interest Formula
Research Lesson	Calculate the saving needed to buy your 1st car
4	Calculating the cost of borrowing money
5	Using Savings v Getting a Loan and implication of both
6	Investigating the Amortisation formula using the Sum of a series
7	Deriving the Amortisation Formula
8	Financial Advice Task

8. Goals of the Research Lesson:

a) Mathematical Goals

- Students will understand how to model investments using a geometric series.
- Students will have access to a tool to aid good financial decisions

b) Key Skills and Statements of Learning

It is intended that this lesson will help students to promote the following key skills, as identified in the Leaving Certificate Mathematics Syllabus,

Critical and creative thinking

- Engaging in investigations
- Critically evaluating information
- Solving problems in a variety of ways

Communicating

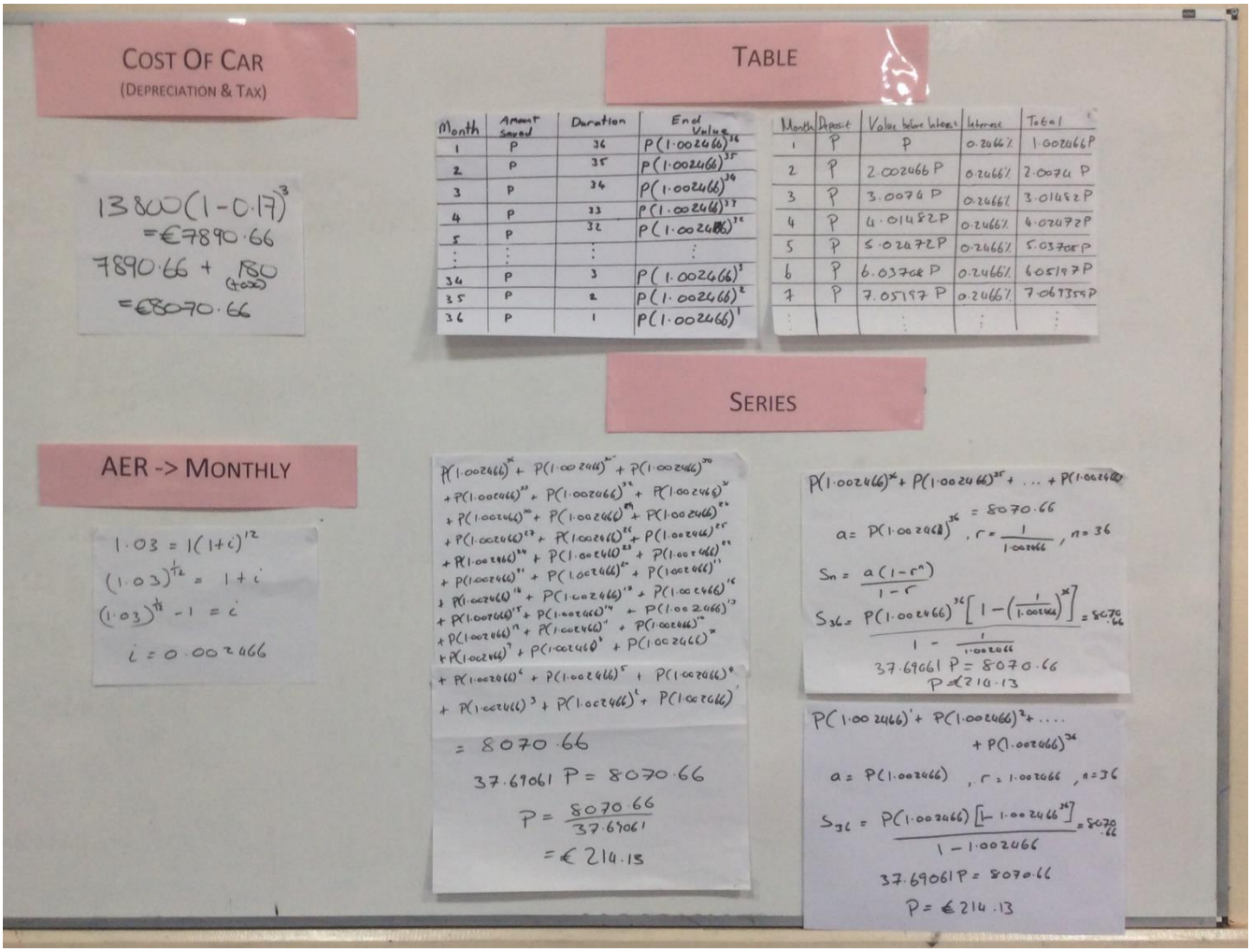
- Discuss approaches to solutions
- Consider and listen to other viewpoints
- Communicate findings to an audience

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction & Posing the Task (5 Min) Distribute the problem	Hand out printed copy of problem. Project on whiteboard. Distribute A3 work sheets	(a) Allow students time to read through the problem. Assess for understanding through questioning. (b) Note/check students understanding or misconceptions in relation to years/time.
Student Individual Work (20 min) Convert APR to monthly rate/ apply the depreciation to the current cost of the car. Use future value formula to work out 1 correct term. Use a table to work out each monthly instalment. List all 36 instalments. Use the sum of a series to determine the amount needing to be saved each month.	**Early finishers** – encourage to investigate how many ways the student could approach the problem. **Scaffold Questions** – Can you use an alternative strategy to find a solution? Refer to identifying key words, applying prior knowledgeable, remind them of prior knowledge. **Incorrect solutions** – Allow wait time. Is there another approach you could take to verify your solution? Encourage them to verbalise their mathematics. Encourage them to work backways to ensure results make sense.	(a)Check that students are actively engaged with the activities through observing their work. Ensure students are verbalising (writing) explanations justifying their work. Record student's responses for board work. - (b)Circulate the room noting student work.

Ceardaíocht /Comparing and Discussing 1. Convert APR to monthly rate/ apply the depreciation to the current cost of the car 2. Use future value formula to work out 1 correct term 3. Use a table to work out each monthly instalment 4. List all 36 instalments 5. Use the sum of a series to determine the amount needing to be saved each month If time allows continue to extension of problem.	How did you calculate the cost of the car in three years? Do you think this is a realistic figure? How many payments are needed? Would there ever be a situation where there would be a different number of payments? Discuss these leading to 35. For those that found one payment – what does P represent? Now you have calculated all the payments. Is there a more efficient way to do this? (Would you have this long in an exam to complete a question?) Can you identify a pattern? How could this question be altered/changed? (Students create their own situation for homework)	(b)Record possible misconceptions surrounding AER and monthly rate. (a)Was depreciation and tax included for the cost of the car?
Summing up & Reflection Refer to earlier Finance options and how they relate to the students.	Look at board work describe and summarise the process. Students record key learning points.	

10. Board Plan



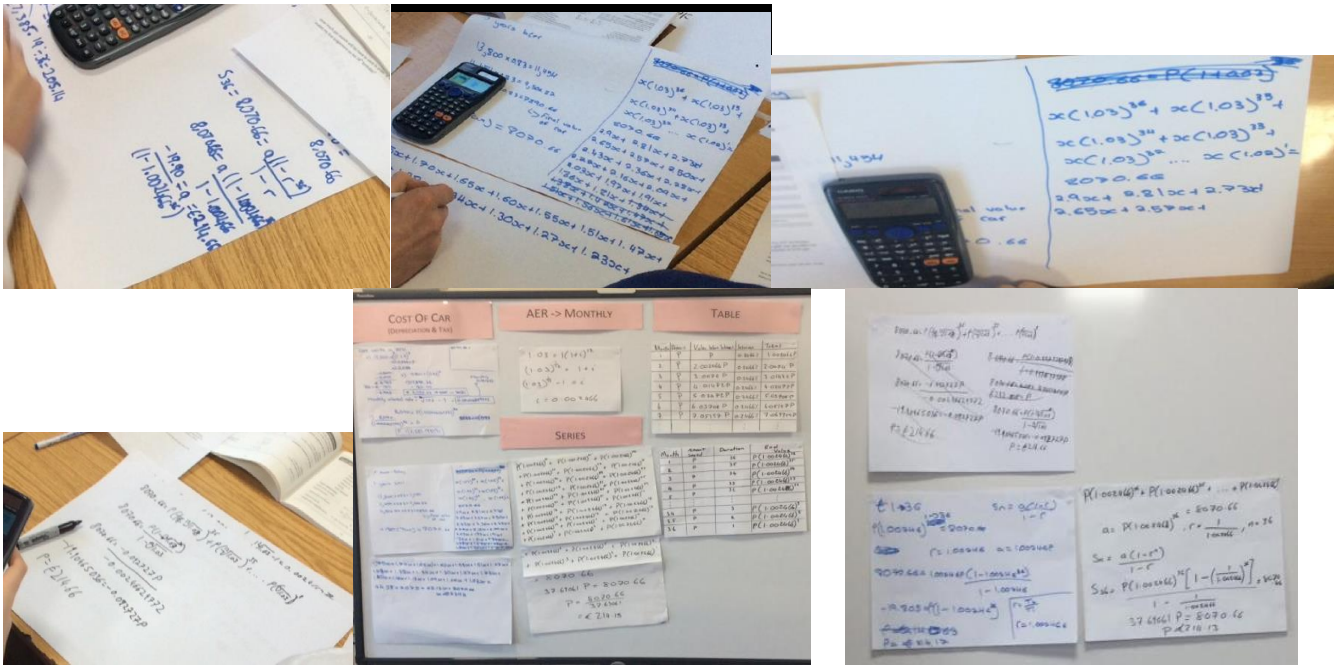
11. Evaluation

a) Students will deal with depreciation, tax and converting APR rate.
b) Students will work out a timeline of payments
c) Students will use future value formula to work out 1 correct term
d) Students will use a table to work out each monthly instalment
e) Students will list all 36 instalments
f) Students will recognise and use the sum of a series to determine the amount needing to be saved each month

1. Did the problem produce the expected responses? (table, list, series)
   There were some difficulties around the depreciation and converting the interest rates. This meant that time had to be taken to address this part of the problem and then allow students to return to the bulk of the problem. During the second attempt at the problem most of the expected responses were attempted although there were some mistakes within these approaches. No students used a table to organise their work which was interesting as we would often teach through making a table.
2. Did the lesson promote active discussion among students?
   Students engaged in good discussions, particularly surrounding the number of payments, the lack of efficiency in listing all the terms and the misconceptions around

the interest rates. There was an interesting discussion between students around what an interest rate to the power of 0 would signify which was teased out in the Ceardaíocht session to address the start and end terms of the series.

3. Were students comfortable with the creation of the timeline of payments?
   Students did not struggle with creating the series, even though initially they did not recognise that was what they were doing. They chose the correct number of payments and applied the interest rate for each month appropriately. What were the most common approaches?
   The most common response was to manually calculate each term, using x for the amount invested and multiplying by the interest rate over the relevant number of months. Towards the end of the problem most students had identified that you should use the sum of a series formula.
4. Was there anything particularly interesting that a student said or any insight a student identified?
   When discussing writing out the series students observed "*the calculator can't even process it"* and "*rounding is an issue"* and that it seemed like "*the correct approach but takes too long".* Another student asked "*Can you make a function?"* Some students re-ordered the series to make it easier and introduced that concept to their peers through Ceardaíocht. One student observed that if you estimated the answer it was a good way to verify the solution was realistic.



Reflection on the Lesson

This lesson went very well. Students were engaged from the beginning and the question challenged students of all abilities within the class.

We hoped that students would use different methods in approaching the problem through lists and tables leading to identifying and creating a geometric series. As this was a text heavy question, it was hoped that students would break down the information into relevant parts. All students approached the question in this way, however, many missed out on details such as depreciating the tax, ignoring the tax, or failing to convert the interest rates. After an initial discussion students were given more time to alter their methods and figures where appropriate.

During the lesson students displayed an ability to make sense of their answers and self-evaluate/reflect. Some students found one payment and were able to recognise that an error had occurred but were unsure of where. This allowed the teacher an opportunity to discuss the time period which deepened student understanding of the process over a 3-year period; '*This is if I make one savings installment but I can spread it out further'* (Student B).

A number of students created a list of payments and added them together. Once this was shown to the class many of the remaining students observed that while the solution was fine this was not time effective, – '*there has to be a better way to do this'*. At the end of the list students were able to recognize that you could apply the sum of a geometric series formula to evaluate the answer instead of the manual calculation of each term in the list.

In the post lesson discussion, the team identified that incorrect solutions should have been addressed in more detail to ensure misconceptions were eliminated. At the end of the class, students could understand the intended solution and make sense of the question, however, some were still unsure as to where they went wrong and how their own answers did not work. If we were to complete the lesson again, these solutions would have been dealt with in more detail to deepen understanding.

After the lesson, the teacher noted that the homework/extension exercise added an extra step too early. Students needed a recap of the material done in class and another attempt at practicing this before advancing on to the first extension. Once they had this practice, the remaining extension exercises were completed within a forty-minute period. This enabled the financial maths topic to be completed in depth in eight forty minute periods. The teacher also noted how the 'fear' of the topic previously encountered by students of past years, was not present within this group. As a result, we have adjusted our planning to complete financial maths in 5th year as part of sequences and series instead of treating it as a separate topic. Hopefully, this will allow students to see the connection between the topics.

The reflection sheet from students largely showed enjoyment of the lesson and a positive learning experience, '*after today, everything is a lot easier to understand as we fully broke the question down and analysed it*'.

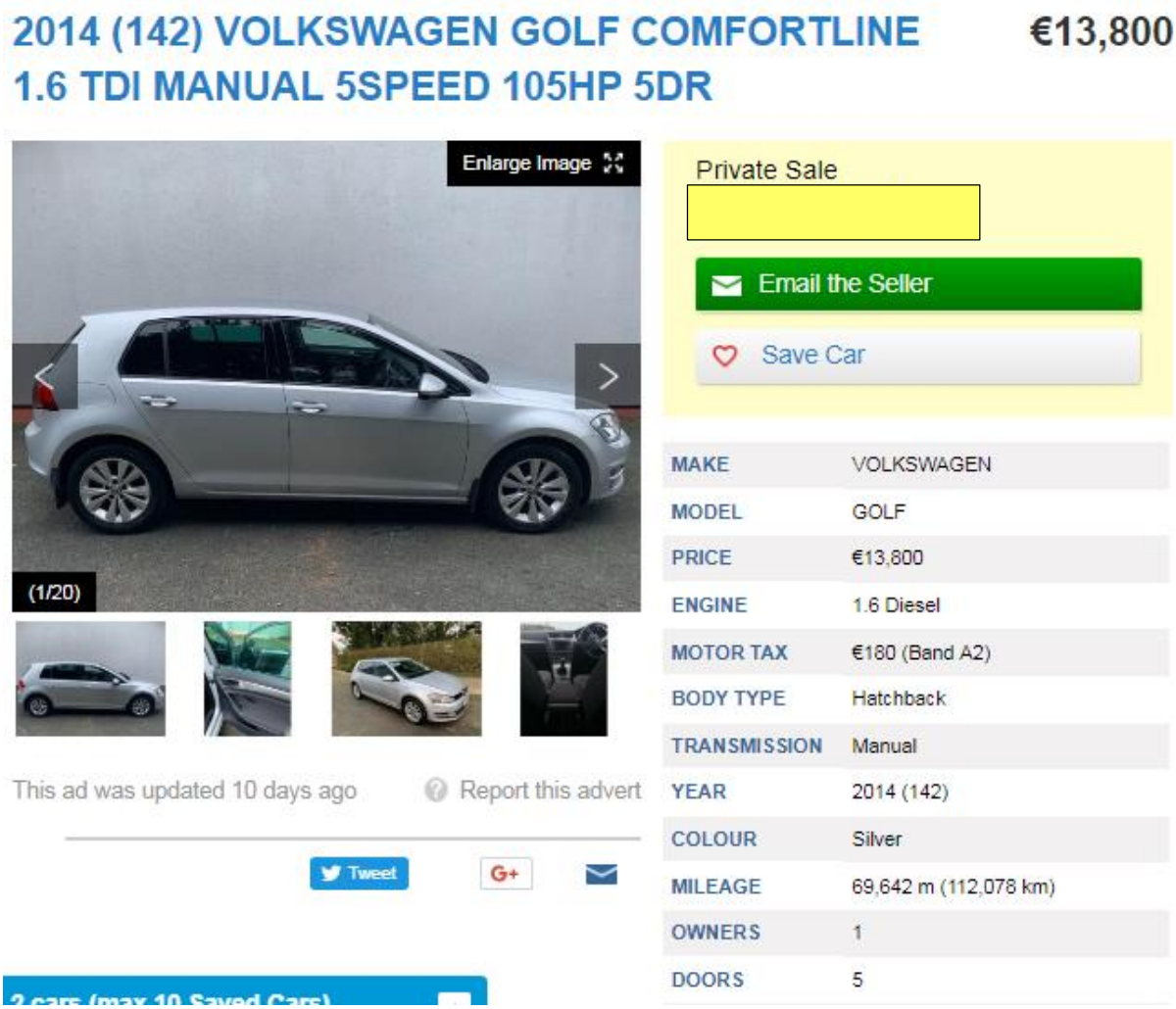
Reflection on the Lesson Study Process

As a result of engaging in the Lesson Study Process we have been afforded an opportunity to reconsider the way we plan as a department. Once the process was completed team members remarked how they '*now see the benefit of lesson study'*. Along with the JCT training days which we completed concurrently with lesson study, our approach and thinking in relation to planning has changed. We are planning units and schemes of work as a department and the focus is now on the needs of students and coherence of the syllabus as opposed to the layout of books. For example, when teaching linear patterns, once the formula is created we are planning to move into functions, algebra and coordinate geometry of the line, as a unit.

The lesson study process afforded us an opportunity to reflect on students learning and on the teaching and learning goals we share as a department. It is rare we have time to meet as a full group and spend time considering best practice and how to maximise student's engagement with a topic. The opportunity to observe how students approach a problem proved to be a valuable experience offering insight into the way students approach a problem.

We will continue with the lesson study process both formally and informally over the coming months and we hope to incorporate it into our planning for non-exam classes also. We already are adapting this unit of learning for lower year groups. As we have now completed the process in full, we have gained more confidence in our planning and are more aware of the goals we want to achieve as a department within our school. Through spending the time together, we have strengthened our collaboration practices and have started to build a strong framework which we hope to continue to improve on for many years to come.

Lesson Question



On the 1st March it is Alex's 18th Birthday. Alex wants to buy a car the day after his 21st birthday. Alex's parents have agreed to pay for his fist year's insurance as a 21st birthday gift. He decides he would like a 2014 Golf TDI equivalent to the one above. Depreciation for this model is 17% per annum. He opens a bank account which will give him an AER of 3%.

How much per month will he have to save in order to afford the car, and the first year of tax, if he makes his first lodgment on his 18th birthday?

Extension (I):

2014 (142) VOLKSWAGEN GOLF COMFORTLINE 1.6 TDI MANUAL 5SPEED 105HP 5DR

€13,800

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Save Car

MAKE	VOLKSWAGEN
MODEL	GOLF
PRICE	€13,800
ENGINE	1.6 Diesel
MOTOR TAX	€180 (Band A2)
BODY TYPE	Hatchback
TRANSMISSION	Manual
YEAR	2014 (142)
COLOUR	Silver
MILEAGE	69,642 m (112,078 km)
OWNERS	1
DOORS	5

After 1 year Alex gets a new job and believes if he increases his monthly deposit he can purchase the car the day after his 20th birthday instead. What would this new monthly deposit be?

Extension (II):

2014 (142) VOLKSWAGEN GOLF COMFORTLINE 1.6 TDI MANUAL 5SPEED 105HP 5DR

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MOTOR TAX	€180 (Band A2)
BODY TYPE	Hatchback
TRANSMISSION	Manual
YEAR	2014 (142)
COLOUR	Silver
MILEAGE	69,642 m (112,078 km)
OWNERS	1
DOORS	5

After one year Alex goes on Erasmus for 6 months. If he stops payments for this time period, but still wishes to purchase the car the day after his 21st birthday, what will be his monthly deposit for the remainder of the time.

Extension (III):

2014 (142) VOLKSWAGEN GOLF COMFORTLINE 1.6 TDI MANUAL 5SPEED 105HP 5DR — €13,800




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Save Car

MAKE	VOLKSWAGEN
MODEL	GOLF
PRICE	€13,800
ENGINE	1.6 Diesel
MOTOR TAX	€180 (Band A2)
BODY TYPE	Hatchback
TRANSMISSION	Manual
YEAR	2014 (142)
COLOUR	Silver
MILEAGE	69,642 m (112,078 km)
OWNERS	1
DOORS	5

On the 1st of March it is Alex's 18th Birthday and he wishes to purchase the above car. His parents have promised to pay the deposit of the car which is 25% of the value. Alex decides to take out a loan for the remainder and the first year's tax. The bank has offered him a loan at 3% APR.

What would Alex's monthly repayments be?